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Corrigendum

Corrigendum to "A comparison between a 3D discrete model and two homogenised plate models for periodic elastic brickwork" [International Journal of Solids and Structures 41 (2004) 2259–2276]

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In a previous work (Cecchi and Sab, 2002) the authors obtained in an analytical form the expression of flexural homogenised constants in the case of running bond masonries under the hypothesis of rigid blocks connected by elastic mortar interfaces. An error occurs in the expression of the homogenised flexural constants (the relative correction is reported in an errata corrige of the paper); hence the original paper referenced above reports this error in the numerical experimentation when the continuum homogenised plate model is compared to the 3D discrete model. In this corrigendum, a correction is reported also for the shear constants identified in Section 3. For simplicity, the numbers of sections, figures, and equations here reported are the same as those of the above-mentioned original paper.

3. The Reissner–Mindlin plate model

The elastic constants $D^F_{\alpha\beta\gamma\delta}$ which relate the plate bending tensor $(M_{\alpha\beta})$ to the curvature tensor $(\chi_{\alpha\beta}) = (-U^{LK}_{3,\alpha\beta})$,

$$M_{\alpha\beta} = D^F_{\alpha\beta\gamma\delta}\chi_{\gamma\delta}, \quad \alpha, \beta, \gamma, \delta = 1, 2, \tag{30}$$

were identified by Cecchi and Sab in an erratum as follows:

$$D^F_{1111} = \frac{\partial^2 W}{\partial \chi^2_{11}} = \frac{t}{12}\,\frac{\left[4K'\frac{e^h}{a} + \frac{b}{a}K''\frac{e^v}{a}\right]t^2 + \frac{b}{4a}\frac{e^v}{a}K''b^2}{4\frac{e^h}{a}\frac{e^v}{b}}, \tag{31}$$

$$D^F_{1122} = 0, \tag{32}$$

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$$D^F_{2222} = \frac{\partial^2 W}{\partial \chi^2_{22}} = \frac{t^3}{12} \frac{K'}{\frac{e^h}{a}}, \tag{33}$$

$$D^F_{1212} = \frac{\partial}{2\partial \chi_{12}} \frac{\partial W}{2\partial \chi_{12}} = \frac{t}{192} \frac{K''\left(\frac{4e^h}{a}(a^2+t^2) + \frac{4e^v}{b}\left(\frac{b^2}{4}+t^2\right)\right) + K'\frac{b}{a}\frac{e^v}{a}t^2}{\frac{e^h}{a}\frac{e^v}{b}}. \tag{34}$$

It must be noted from Eq. (31) that D^F_{1111} presents an additional contribution due to the term $\frac{b}{4a}\frac{e^v}{a}K''b^2$. Besides, the only correction in Eq. (34) is the $b^2/4$ term instead of b^2. From a numerical point of view the D^F_{1111} increases while the D^F_{1212} decreases.

A Reissner–Mindlin orthotropic plate model is proposed to take into account shear effects. The bending elastic constants must be the same as those of the Love–Kirchhoff model (30)–(34) because these two models are asymptotically equivalent when the ratio t/L goes to zero. In a Reissner–Mindlin orthotropic plate model, the shear elastic constants ($F_{\alpha\beta}$) relate the shear stress vector (Q_α) to the shear strain vector ($U^{RM}_{3,\alpha} + \phi_\alpha$) as follows:

$$Q_1 = F_{11}(U^{RM}_{3,1} + \phi_1), \qquad Q_2 = F_{22}(U^{RM}_{3,2} + \phi_2), \qquad F_{12} = 0. \tag{44}$$

The identification of F_{22} may be obtained from (14) as reported in the original paper. On the contrary, the Reissner–Mindlin shear constant F_{11} reported in the original paper is not correct. In fact, if a periodic shear force in the direction 3 is taken into account along the vertical cross section for $B_{i,j}$ centre, then the contribution of the horizontal joints must be taken into account as follows:

$$Q_1 = \frac{1}{2a}\left([B_{i+1,j+1} \to B_{i-1,j+1}] + [\text{right side of } B_{i,j} \to \text{left side of } B_{i,j}]\right). \tag{I}$$

The equilibrium of the right side of $B_{i,j}$ gives

$$[\text{right side of } B_{i,j} \to \text{left side of } B_{i,j}] = [B_{i+1,j-1} \to B_{i,j}] + [B_{i+1,j+1} \to B_{i,j}] + [B_{i+2,j} \to B_{i,j}] \tag{II}$$

and

$$Q_1 = \frac{1}{2a}\left([B_{i+1,j+1} \to B_{i-1,j+1}] + [B_{i+1,j-1} \to B_{i,j}] + [B_{i+1,j+1} \to B_{i,j}] + [B_{i+2,j} \to B_{i,j}]\right). \tag{III}$$

Hence using (14) and (21), the normalised shear force is

$$\begin{aligned} Q_1 &= -\frac{1}{2a}\left(\frac{\partial W^{1,1}}{\partial u_3^{i-1,j+1}} + \frac{\partial W^{1,-1}}{\partial u_3^{i,j}} + \frac{\partial W^{+1,+1}}{\partial u_3^{i,j}} + \frac{\partial W^{+2,0}}{\partial u_3^{i,j}}\right) \\ &= \frac{K''t}{2e^v}\left(u_3^{i+1,j+1} - u_3^{i-1,j+1} + b\frac{\Omega_2^{i+1,j+1} + \Omega_2^{i-1,j+1}}{2}\right) \\ &\quad + \frac{K''}{e^h}\frac{bt}{4a}\left(u_3^{i+1,j-1} - u_3^{i,j} + \frac{b}{2}\frac{\Omega_2^{i+1,j-1} + \Omega_2^{i,j}}{2} + a\frac{\Omega_1^{i+1,j-1} + \Omega_1^{i,j}}{2}\right) \\ &\quad + \frac{K''}{e^h}\frac{bt}{4a}\left(u_3^{i+1,j+1} - u_3^{i,j} + \frac{b}{2}\frac{\Omega_2^{i+1,j+1} + \Omega_2^{i,j}}{2} - a\frac{\Omega_1^{i+1,j+1} + \Omega_1^{i,j}}{2}\right) \\ &\quad + \frac{K''t}{2e^v}\left(u_3^{i+2,j} - u_3^{i,j} + b\frac{\Omega_2^{i+2,j} + \Omega_2^{i,j}}{2}\right). \end{aligned} \tag{47}$$

With (41)–(43) and U_3^{RM} of order 1 and ϕ_α of order 0, it is found that

$$Q_1 = F_{11}(U^{RM}_{3,1} + \phi_1), \tag{48}$$

with

$$F_{11} = \frac{K''bt}{e^v} + \frac{K''b^2t}{4ae^h}. \tag{49}$$

The corrected expression for F_{11} presents the additional contribution $\frac{K''b^2t}{4ae^h}$ to the original expression.

Table 1
Homogenised flexural moduli: Wrong value referred to the original paper and correct value referred to the actual paper

t (mm)	D_{1111}		D_{2222}	D_{1212}		F_{22}	F_{11}	
	Wrong	Correct		Wrong	Correct		Wrong	Correct
120	2.728×10^{10}	3.653×10^{10}	4.126×10^{9}	9.8×10^{9}	8.457×10^{9}	1.375×10^{6}	6.25×10^{6}	1.335×10^{7}
180	9.207×10^{10}	1.059×10^{11}	1.39×10^{10}	2.89×10^{10}	2.69×10^{10}	2.063×10^{6}	9.375×10^{6}	2.003×10^{7}

4. Numerical results: a comparison between the three models

In this section, a comparison between the Love–Kirchhoff model, the Reissner–Mindlin model and the 3D discrete model is conducted on a test case. In the figures from 5 to 9 of the original paper, in the numerical experimentation, no consistent differences in the $e_{\%}$ percent error may be pointed out both for the Love–Kirchhoff and Mindlin–Reissner models. For this reason the above-mentioned figures are not here repurposed.

An explication of this phenomenon may be found in the following remark:

- The deflection of the plate presents as a principal direction the direction 2. In fact, in this direction the plate is more deformable than in direction 1. This condition corresponds to the case of a beam with its longitudinal axis coincident with the direction 2 of the plate. Hence the homogenised constants, consistent for the deflection, are D^F_{2222} and F_{22}. In fact, as shown in Table 1, $D^F_{2222} < D^F_{1111}$ and $F_{22} < F_{11}$. For completeness, in this table, also the wrong values of the original paper by comparison to the actual correct values are reported for two plate thickness $t = 120$ mm and $t = 180$ mm.

Reference

Cecchi, A., Sab, K., 2002. Out of plane model for heterogeneous periodic materials: the case of masonry. Eur. J. Mech. A/Solids 21, 249–268.